

PS 8: Optimal Pricing Competition

Continued:

Your friend runs a firm. After hearing about your experience using big data, she has asked to help her optimize the price of her product. You've helped her analyze pricing in a monopolistic market, but now a competitor is going to be entering the market.

You may turn in one write up for your entire group.

A competitor plans to enter the market. How should your friend change her price in response?

Let's think through the theory. There are many individuals and more than one product.

Assume an individual i gets the following utility from buying product j :

$$u_{ij} = \mu_j - \alpha p_j + v_{ij}$$

where μ_j is the base level of utility the product provides, α is price sensitivity, p_j is price, and v_{ij} is an idiosyncratic shock—how much i in particular likes product j .

If v_{ij} is drawn from a particular type of distribution, a generalized extreme value distribution, then it is easy to solve for the overall demand for product j :

$$D_j = \frac{\exp\{\mu_j - \alpha p_j\}}{1 + \exp\{\mu_j - \alpha p_j\} + \exp\{\mu_{-j} - \alpha p_{-j}\}}$$

Where $-j$ represents the other product. (We've assumed that not buying either good—the outside option—provides utility of zero, which leads to the quantity "1" in the expression.¹)

In particular note that we don't need to keep track of the idiosyncratic shocks v_{ij} .

1. **Derive Mathematically:**
 - a. How does D_j change as μ_j increases?
 - b. How does D_j change as the price of good j increases? How does it change as the price of the other good increases?
2. Develop a procedure to find the profit maximizing price p_j as a function of p_{-j} . (Marginal cost should enter your profit function.) A precise description of an algorithm you would implement is sufficient for this part.
3. Your procedure depends on p_{-j} . How should you think about your competitor's price?

¹ Note that $\exp\{0\} = 1$

For this coming week, your group will be matched with another group at random which will compete with you over customers in one market. You are in an industry where the marginal cost of production changes rapidly, and is the same for all firms.

Each day, you will be asked to submit a price by midnight; if you do not submit a price we'll assume you keep the same price as the previous day.

We'll compute sales figures for that day based on submitted prices, and report these back to you by 5pm. We'll also report an analyst's forecast of the next day's marginal cost. You can use these sales figures to set your price p_j .

Your goal is to maximize profit earned over the course of the week; the most profitable group will be the winner. We'll discuss the results in the next class.

You can use the worksheet below to organize your response.

Parameters (held constant throughout the exercise)

Parameter	Value
$\mu_j = \mu_{-j}$	5
α	0.5

Worksheet

Date	Ex-Ante		Ex-post (<i>revealed by 5pm following day</i>)		
	Analyst's Forecast of Marginal Cost	Your Price (p_j) (<i>submit by midnight</i>)	Opponent's Price (p_{-j})	Realized Marginal Cost	Your profits
Wednesday	0.5				
Thursday					
Friday					
Saturday					
Sunday					
Monday					